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A Logico-mathematic, Structural Methodology: Part II, Experimental Design and Epistemological Issues

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In this first of two companion papers to a logico-mathematic, structural methodology (Haskell, 2003, this journal), a meta-level analysis of the non metric structure is presented in relation to critiques based on standard experimental, statistical, and computational methods of contemporary psychology and cognitive science. The concept of a non metric methodology is examined as it relates to the epistemological and scientific goals of experimental, statistical, and computational methods. While sharing in these goals, differences and similarities between the two methodological approaches are outlined. It is argued that typical experimental methods are not sufficient to extract and validate semantic information in verbal narratives. It is further suggested that a logico-mathematic, structural methodology can yield invariant law-like cognitive processes by careful methodological control of the specific case — instead of those found by current methods that produce "laws" based on statistical frequency. Lastly, the issue of experimental manipulation in relation to the logico-mathematic, structural methodology is examined.

This first of two companion papers to a logico-mathematic, structural methodology (see Haskell, 2003, this journal) addresses issues involved in critiques directed at the logico-mathematic method and its findings of unconscious cognition and complex linguistic and numeric referents as given in narratives. It is suggested that previous critiques are inappropriately based on experimental, statistical, and computational methods that do not apply to logico-mathematic, structural methodologies.[1] Throughout the development

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[1]For current purposes, this paper focuses on issues applicable to experimental and statistical methodology, not computational approaches. While there are significant differences between

of the logico-mathematic method, elements of two disciplines have been utilized as models in explaining the method and its yield of unconscious or subliteral ($S_{ub}L_{it}$) phenomena.[2] The first, mathematics, is characterized not primarily by its measurement precision, but by its logical consistencies, internal transformations, and inferential operations. The second is linguistics. Both are essentially structural frameworks which require neither statistical tests, sampling procedures, nor experimental design. Nevertheless, over the past several years reviewer critiques have invariably cited the fact that $S_{ub}L_{it}$ findings have been neither subjected to experimental design nor to statistical test and sampling procedures.

The importance of statistical sampling has been so much a part of experimental psychology that not to include sampling procedures in research is considered an egregious omission that automatically disqualifies any variant methodology as scientifically legitimate. Accordingly, the present paper will examine concerns that the logico-mathematic methodology diverges from the accepted practices of (a) experimental design, (b) statistical tests, and (c) sampling procedures, and will explore (d) epistemological differences inherent in experimental and statistical methods on the one hand and the logico-mathematic methodology on the other.

The methodological issue addressed is similar to that in linguistics. Linguistics did not develop its theories of syntax using experimental designs and statistical tests. Rather, the study of linguistics involves a structural, inferential, and lawful system of relations. The use of statistics is appropriate when attempting to show the *magnitude* of the relation between two variables or the degree of variance among variables. Statistics is not appropriate for studying structurally lawful mechanisms. Its proper use is in estimating parameters or in testing for the probable presence or absence of relationships when the observables are regarded as random or at least as highly variant. Sampling and other statistical methods, then, do not apply where lawful regularity is presumed to exist. In dealing with linguistic and with verbal language (speech), lawfulness is assumed. For example, the concept of speech parts and their combinations such as nouns, verbs, adverbs and so on is regarded as neither random nor merely probable but as lawful to language. Likewise, the concept of the *meaning* of a sentence constructed with parts of

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experimental, statistical, and computational methods, e.g., procedural v. inferential v. algorithmic, etc., I have grouped all three on the basis of their (a) wide acceptance in psychological research, and (b) treatment of data as context independent and content empty, i.e., as syntactic (abstract) v. semantic relations. Such approaches are the consequence of what are thought to be pragmatic limitations concerned with contextual complexity. Fodor (1980) has labeled this pragmatic approach (apparently after Carnap) *methodological solipsism*.

[2]For brevity and ease of exposition, “logico-mathematic, structural methodology” will be textually referred to as “logico-mathematic method.”

speech is not regarded as random or probable (from the point of view of the speaker) even though the sentence construction itself may show some variation. The point here is that experimental/statistical methods are neither necessary nor sufficient for investigating complex linguistic processes just as they are not relevant in linguistics or in mathematics where inferential and logical procedures are used for analysis and validation.

The purpose of these two companion papers, then, is to clarify the logico-mathematic method, and to provide a basis for researchers to recognize that the methodology and its $S_{ub}L_{it}$ findings belong firmly in the cognitive sciences and not relegated to psychoanalytic theory, literary theory, or discourse analysis as typically occurs with material involving complex and unconscious linguistic referents.[3] The findings are considered too significant — to both cognitive theory and psycho-linguistic theory — to be relegated to these domains.

Experimental, Statistical, and Logico-mathematic Design

Contemporary psychology and cognitive science continue to be firmly committed to experimental and statistical methods. Historically, for understanding and isolating variables and for assessing statistical variance — as well as for both pure and applied productivity over a broad range of phenomena — most researchers would agree that the commitment to experimental and statistical methods has not been misplaced.

Despite this broad consensus, a troubling question remains: Are experimental and statistical methods *sufficient* to investigate *all* phenomena in psychology? While researchers with a strict adherence to experimental and statistical methods would likely agree that non experimental and statistical methods do not yield scientifically valid findings, those who hold a less strict adherence to these methods as the only valid tools available answer “no” to this question for certain kinds of data. In short, those who hold a strict commitment to experimental and statistical methods consider data manipulated with other methods as not properly belonging to a valid methodology. Indeed the coherence holding the increasingly variegated field of psychology together is attributed to an adherence to experimental method (Darley, 2001). This strong view, or strict commitment, to experimental and statistical methods tends to dominate APA and APS journal publication criteria.

Let the thesis here be clear: the problem being addressed is not with experimental and statistical methods per se but with the widespread exclusionary stance toward non experimental and statistical methodologies. Nor is it being suggested that experimental and statistical methods are always inap-

[3]For brevity and ease of exposition, the term “unconscious” will be used in place of the more contemporary research-based conceptualization of a “cognitive unconscious.”

propriate for applications to data that otherwise require additional methodologies (see more below). Though the majority of phenomena may be amenable to experimental and statistical methods, such methods are not sufficient to extract and validate the kind of semantic information in the verbal narratives presented by Haskell (2003, this journal). In this regard, Whitehead (1929) has stated that "Every science must devise its own instruments" (p. 16). Accordingly, this paper examines the concept of a non experimental methodology in relation to the epistemological and scientific goals of experimental and statistical methods.[4]

The split between those holding an exclusive adherence to experimental and statistical methods and those who hold a less exclusionary view has deep historical roots which continue to divide the field. It is not the purpose of this paper, however, to revisit old arguments regarding the various definitions of what constitute rigorous scientific methods other than to posit experimental and statistical methods as here described, and the logico-mathematic method explicated in Haskell (2003, this journal) as an operational exemplification. Nevertheless, it will be necessary to contrast experimental and statistical methods with the logico-mathematic method.

The logico-mathematic method presented in Haskell (2003) adheres to what is generally considered the five essential characteristics of scientific methodology: (1) the systematic gathering and (2) manipulation of empirical data, with (3) rigorous procedures for the control of that data and (4) the controlled testing of hypotheses, (5) all of which are involved in falsification criteria.[5] Accordingly, this paper does not generally support the use of non metric or non quantitative methods per se as, arguably, most do not conform to this fundamental set of criteria. While many qualitative methods utilize statistical analyses, and while others are capable of the systematic gathering of empirical data, most are legitimately seen as neither amenable to rigorous manipulation and control of data, nor to rigorous hypothesis testing leading to potential falsification — for example, grounded theory (Glaser and Strauss, 1967) and hermeneutic methods (Packer, 1985) — though such approaches

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[4]Similar to Whitehead's dictum, the eighteenth century Italian philosopher Giambattista Vico in his *New Science* (1727/1948) argued that "Doctrines must take their beginnings from that of the matters of which they treat" (p. 49), by which he meant — for example — that to understand poetry, methods appropriate to that subject matter must be utilized. Clearly, experimental and statistical methods are not sufficient for such subject matter.

[5]All too often, when the term "qualitative" precedes the term "method," it denotes a modicum of systematicness, but little to no controlling, testing, and falsifying procedures. While the logico-mathematic method is non metric, I do not consider it "qualitative" in the common sense of the term. Rather it is systemically structural in the sense of algebra, geometry, and topology.

may generate hypotheses. Finally, the logico-mathematic method is one that can yield invariant "laws" — ones not based on statistical frequencies or generalization as in most of cognitive science.

Galilean v. Aristotelian Methodological Approaches

Kurt Lewin (1931) in his now seminal but little cited paper on the conflict between what he conceptualized as Aristotelian versus Galilean research in psychology, addressed in significant measure the problematic of statistical methods. Lewin developed a field theory of human behavior modeled on physics and on topology — a qualitative branch of mathematics — the well-known notation for which is $B = f(P, E)$, where B (behavior) is a function (f) of P (person) and E (environment).[6] Aristotle explained phenomena on the basis of their nature: that is, rocks fall to the ground because it is in their nature to do so. Most importantly, however, Lewin objected to the Aristotelian notion of "lawfulness" based on statistical frequency of occurrence. According to Lewin, modern psychology essentially adopted an Aristotelian mode of explanation with its emphasis on statistical analyses. But Lewin did acknowledge differences between modern psychology and the Aristotelian and Galilean approaches. For example, while psychology is experimental, the Aristotelian approach was not. Nevertheless, Lewin's point is that, like Aristotle, psychology still tends to explain lawfulness as frequency, with statistical correlations yielding group probabilities or central tendencies. In contrast, a Galilean approach yields invariant laws by a thorough and careful examination of a single case, thus describing invariant individual lawfulness.

Classically, "laws" in the physical sciences are invariant (under specified conditions), as opposed to generalizations based on statistical probabilities.[7] For example, Galileo's well-known discovery of the "law of falling bodies," as indicated in the formula $s = \frac{1}{2} gt^2$, states that the distance an object falls from a resting position is proportional to the square of the time the object is in motion. Galileo's experimental findings did not result in statistical conclusions. Unlike laws in psychology, as Lewin (1931) quipped, "The law of falling

[6]Despite adopting this model from physical science, Lewin's theory and method were in fact — though not necessarily in theory — non metric. His field theory never caught hold. Though Lewin's field theory had a general impact in social psychology, it is virtually ignored in North American psychology. It remains of interest, however, in Europe — reflecting another historical difference between a European orientation which leans toward theory and an American orientation around experimental and statistical methods (see Berlyne, 1968; Boring, 1928; Hall, 1990).

[7]It perhaps needs to be made clear that while psychological experiments can be designed to show causal (and hence, "lawful") relations, in practice demonstrating causal relations is exceedingly rare.

bodies . . . does not assert that bodies *very frequently fall downward*" (p. 150, italics added).

It is this transition to a Galilean mode of identifying invariant laws that, according to Lewin, characterizes a mature science. As Lewin (1931) explained,

> What is now important to the investigation of [Galilean] dynamics is not to abstract from the situation, but to hunt out those situations in which the determinative factors of the total dynamic structure are clearly distinctly and purely to be discerned. Instead of a reference to the abstract average of as many historically given cases as possible, there is a reference to the full concreteness of the particular situations. (p. 166)

Lewin's view was that psychology should be about discovering invariant "laws" of human behavior, just as Galileo did in astronomy and physics by carefully examining the specific case. The non metric logico-mathematic method addressed in this paper lends itself to Lewin's lament. Just as the linguistic structure of language is characterized by lawful relationships, so is the structure of sub-literal language (Haskell, 2003).

Today, as in Lewin's (1931) time, however, a researcher who considers standard experimental and statistical methods (e.g., frequencies and correlations) as not appropriate for studying all human behavior, "usually encounters a weary scepticism" (p. 155), or is seen as exhibiting a "maudlin [over] appreciation" (p. 155) of the individual case. Moreover, in Aristotelian terms "*qualitative properties are considered the exact opposite of lawfulness*" (p. 155, italics added). Additionally, to suggest the existence of psychological "laws" of human behavior in the Galilean sense tends to be seen by many as either naive or as reflecting an outdated philosophy of science. Indeed, to talk about "laws" in psychology is seen as somewhat grandiose or at least premature. Such attitudes have not significantly changed since Lewin's cogent analysis. There are exceptions, however, as Lewin notes; for example the "laws" of sensory psychology or psychophysics.

As Lewin further pointed out, "The propositions of modern physics, which are often considered to be 'anti-speculative' and 'empirical' unquestionably have in comparison with Aristotelian empiricism a much less empirical, [and] *a much more constructive character*" (p. 150, italics added). Along with a non metric lawful structure, it is this *constructive* character — in contradistinction to the phenomenally given character of North American empirical psychology — that is addressed here in relation to the logico-mathematic method.

Though separated by nearly sixty years from Lewin, Shepard (1987) has suggested at least one psychological "law" in the Galilean sense: it is the universal law of generalization. Indeed, as Shepard (1994) later suggested, psychological science need not limit itself to descriptions of observed empirical regularities (i.e., frequency based and phenomenological data), but:

> aspire to a science of mind that, by virtue of the evolutionary internalization of universal regularities in the world, *partakes of some of the mathematical elegance* and generality of theories of that world. The principles that have been most deeply internalized may reflect quite abstract features of the world, based as much (or possibly more) in geometry, probability, and *group theory* as in specific, physical facts about concrete, material objects. (p. 26, italics added)

It should be noted, however, that Shepard's work belongs largely to sensory psychology and psychophysics. Nevertheless, he is suggesting non statistical but mathematical structures to thought and behavior. The logico-mathematic method can yield a kind of lawful, mathematical elegance. Indeed, Haskell and Badalamenti (2003) have recently demonstrated what Haskell had suspected for some time: that a series of $S_{ub}L_{it}$ stories with numeric references to "3s" (see Haskell, 2003, this journal) forms a permutation group; more specifically an Abelian algebraic semi-group. And it appears that there may be further mathematical structures involved.

Finally, in examining experimental design in the manipulation of implicit (unconscious) cognition research, Steele and Morawski (2002) correctly observe that "Through *aggregate techniques*" [read statistical], "the subject of psychology became generic, yielding generalizable, useful and, not least, quantitative knowledge" (p. 47, italics added). And consequently, as Danziger (1990) suggests, nomothetic or aggregate methods "made it seem eminently reasonable to ignore the settings that had produced the human behavior to be studied and to reattribute it as a property of the individual-in-isolation" (p.186).[8] The logico-mathematic method is derived in part from a contextual field analysis (see *[2.] Contextual Procedures*).

Antecedents and Precursors of the Logico-mathematic Methodology

The non metric structural methodology developed by Haskell (2003, this journal), though related to what is known as structuralist methods, is not derived from them. During the 1970s and 1980s, a wide range of methods called structuralist were in vogue in many disciplines. In fact, structuralism was considered inherent in the infrastructure of some fields, and was adopted for a time in yet other fields as both a theory and a methodology. Indeed, structuralism became an umbrella term for a number of related methods and theories.[9] As here described, structuralism — as in mathematics — essentially

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[8]Cited in Steele and Morawski (2002).

[9]It should be noted that structuralism as discussed here bears no relation to the classic structural/functional approach of nineteenth century psychology, e.g., Titchener (1910).

views phenomena as systems of elements characterized by complex, integral transformations.

In psychology, structuralist perspectives are exemplified in the work of Mucchielli (1970), Piaget (1970, 1971a, 1971b), Riegel and Rosenwald (1975), and in certain aspects of psychoanalytic (Freud, 1960) infrastructure. More specifically, structuralist methods have been perhaps best exemplified in anthropology by Levi–Strauss (1963), and in modern linguistics by Chomsky (1968). A structuralist approach also became rather fashionable in literary criticism (see Boon, 1972; Ehrmann, 1966), as well as in other areas of research (see Gardner, 1972; Wilden, 1972). It is mathematics, however, that best exemplifies a structuralist model (see below).

With the exceptions of mathematics and linguistics which provided the initial paradigm for the formulation of structuralist approaches, none of these fields sustained a coherent systemic set of methods or general theory. Though influenced by these structural approaches, the logico-mathematic method presented in this paper and in Haskell (2003, in press) evolved primarily out of a detailed cognitive and psycho-linguistic analysis of empirical verbal data.

Linguistic Structure and $S_{ub}L_{it}$ Analysis

Generally, linguistics as a field of research for investigating *structure* (syntax) and *meaning* (semantics) in verbal productions utilizes neither experimental design nor statistical sampling (the stochastic approach to lexical relationships notwithstanding). As explained above, the relationship of linguistic structure to its referent is presumed to be a natural *lawful* process, not a random or probable event. Again, linguistics, like mathematics, is concerned (as were Lewin, Galileo, and Shepard) with an invariant lawfulness, and not with relationships based on probability or frequency of occurance.

A fundamental difference between most linguistic methods and the logico-mathematic method is that while linguistics examines abstract structures, it does not address derived extended "meaning" in everyday sentences. For example, it is well-known that in the structural linguistics of Chomsky, "meaning" is not part of the research design. Neither does structural linguistics address "metaphorical" meaning. In fact, the dual referents involved in metaphors are considered a "deviant" form of correct usage. Moreover, the field of linguistics in general tends not to deal with how specific individual meaning is constructed and derived from a string of words called a sentence.

Sampling procedures are not relevant in a logico-mathematic methododology, then, because the purpose is not to claim that all or most verbal narratives exhibit logico-mathematic operations, but rather that the form some operations take, as well as their $S_{ub}L_{it}$ referents, under specified conditions, is contextually, structurally, and systemically lawful.[10] In this sense the findings presented are not only similar to mathematical structure but to linguistic analysis and structure. Linguists do not work with random or representative samples; nor did Galileo, nor do contemporary physicists. *As linguistic syntax is the controlling methodology for semantic meaning in language, so is the logico-mathematic method the controlling methodology for understanding $S_{ub}L_{it}$ cognition and referents.*

More compatible with $S_{ub}L_{it}$ language — at least in principle — is Langacker's (1987) formulation of a cognitive grammar, the core claim of which is that language is completely describable in terms of semantic and phonological structures with a symbolic link between them. According to this view, only symbolic or schematized structures are necessary for positing lexical, morphological, and syntactic structure, with each forming a gradation that can only arbitrarily be divided into discrete components. Syntax in this system is generated by schematizations of phonological structures. The point is that language is built upon cognitive processes.

Logico-mathematic and Structural Operations

Operationally the method addressed in Haskell (2003) can be generally exemplified by a linguistic methodology, and by a logico-mathematic order as well as by a general systems theory approach, e.g., Bertalanffy (1963). Structuralist approaches tend to be non metric. Excluding branches like algebra, set theory, and topology, the rest of mathematics is a clear exception. Accordingly, structuralist approaches do not require experimental and statistical or measurement methods.

Essentially, a structure is defined as an integral system of elements and their transformations derived by operations performed on empirical data. Structures of transformations are lawful sets of operations within a system maintained and enhanced by the integral character of its internal operations. In mathematics these operations and transformations do not yield results external to the system; nor do they employ elements external to the structure; mathematical systems are relatively closed systems of elements and their transformational operations. Being an integral system of transformations, then, a structure is not a mere collection of elements, but rather is an integral set. The terms integral and transformation in this definition exclude simple

[10]Classically, as Nagel (1961) clearly shows, a scientific theory or law is "state dependent." That is, it is always situated within a description of its surrounding conditions. In logico-mathematic terms, the concept of state dependent translates to a description of the contextual conditions under which a given operation will occur.

structures in the sense of "form." The structure of a building or a society, for example, is not a structure in the logico-mathematic sense. In contrast, an algebraic structure is the epitome of structuralist thinking.

A mathematical group (and its subgroups) is a structure because it is based on a system of transformations in which each of its subclasses, for example integers, can be reconstructed by reference to any of its elements. Thus, a logical class such as "furniture" is not structured in this sense because the properties of its subclasses, for example a "table," can not lead to the re-construction of other subclasses or to the class as a whole as do numeric structures.

Piaget attempted to demonstrate the logico-mathematic structure in his data. As is well-known, he claimed to show that children develop proto logico-mathematic operations, especially with regard to the development of number. Additionally, but seldom mentioned, Piaget believed he had illustrated that cognitive development in children parallels the three Bourbaki elementary mathematical structures: an algebraic structure, an order or network structure, and a topological structure. Piaget (1971b) contended that his findings suggested that mathematical structures are inherent in cognitive development and are derived from a primitive sensory–motor action (the veridicality of sensory–motor origin of number is not important for this paper).

Analyzing preliterate mythologies, Levi–Strauss (1963) attempted to demonstrate a logico-mathematic structure. He maintained that for any series of cultural myths it should be possible to order the myths into a series of transformations of the same logical type. He stated,

> By systematically using this kind of structural analysis it becomes possible to organize all the known variants of a myth into a set forming a kind of *permutation group*, the two variants placed at the far ends being in a symmetrical, though inverted, relationship to each other. (p. 223, italics added)

Levi–Strauss understood that structural methods

> are, in the social sciences, the indirect outcome of modern developments in mathematics which have given increasing importance to the qualitative point of view in contradistinction to the quantitative point of view of traditional mathematics. It has become possible, therefore, in fields such as mathematical logic, set theory, group theory, and topology, to develop a *rigorous* approach to problems which do not admit of a metrical solution. (p. 26, italics added)

As with Lewin, and with Shepard (above), Levi–Strauss believed it possible to discover the invariant mathematical structures undergirding cognitive processes.

In considerable measure the criticism leveled against Levi–Strauss was the consequence of reviewers not understanding his structuralist epistemology and method. Accordingly, his work was evaluated on the basis of the predominant experimental and statistical epistemology (see below).[11] Thus, critics of Levi–Strauss deny that his methodology succeeded (Leach, 1974), with one possible exception. Levi–Strauss seems to have demonstrated that systems of kinship can be understood as a set of algebraic-like structures. His kinship algebra, as it has come to be called, appears valid and generally accepted, but with questions as to its usefulness (Cargal, 1996).

Where Levi–Strauss's structural method runs into difficulty is in his claim that preliterate myths have an ordered algebraic structure. Indeed, his original formulation is now largely relegated to the history of anthropology. Despite Gardner's (1972) early recognition and proclamation that Piaget's and Levi–Strauss's work represent "the most significant contemporary innovation" (p. xii) in the social and behavioral sciences, structuralism reached a relative dead end in the mid 1980s.

The promise of structuralist methods in the behavioral sciences, thus, never fully developed. As in all of science, researchers must be able to replicate the operations performed on a set of data and to derive the same structures. Other researchers, however, all too often failed to derive the same structures that Levi–Strauss claimed existed between myths and their "variants" or transformations. The problem was — and still is — that without a controlling methodology, mappings of the data lead to widely different "interpretations" of a given set of myths as to their equivalence or invariant structures. The same problem exists when applying structuralist methods to literary works.

The problem is that many researchers like Levi–Strauss, claiming to find logico-mathematic structures and operations, worked with semantic data.[12] Semantic data, however, are not as clearly bounded as numeric data. Since the methodology (Haskell, 2003) discussed here includes a large corpus of numeric references found in narratives, it largely escapes this historical problem that has plagued other structuralist approaches. The extended exemplification of the set of stories with numeric references to "3s" presented in the

[11]Though various aspects of Piaget's and of Freud's theories remain of interest in mainstream psychology (the former more than the latter), it is not because of their constructivistic framework. And while Piaget's cognitive developmental approach remains of interest in developmental psychology, it has been largely recast to be compatible with experimental and statistical methods, losing much of the essential logico-mathematic structure of Piaget's genetic epistemology. This simplification is nowhere more evident than in most forms of the resurgent Piaget/Dewey-inspired constructivist educational theory (e.g., Fosnot, 1992).

[12]While many developmental psychologists have adopted Piaget's logico-mathematic constructivist approach, few have taken seriously the full implications of his genetic epistemology as exemplified in what is here considered his three major theoretical works (1970, 1971a, 1971b). Despite Piaget being adopted by developmental psychology, Piaget viewed himself a genetic epistemologist.

methodology (see Haskell, 2003, this journal) shows that the numeric references are all variants of a set which form a type of (mathematically constructive) algebraic group (see Haskell and Badalamenti, 2003). Thus, unlike analyzing semantics as Levi–Strauss did with primitive myths, and as structuralists attempted with literary works, the set of numeric stories is relatively bounded and concrete and can be more definitively mapped onto the membership composition of the narrative situation as well as tracked throughout the entire protocol. Numeric references are therefore not as problematic for researchers to reach agreement on and thus to derive the "same" referents and structures. Given such findings, by implication or inference, it is reasonable to assume such a cognitive structure would undergird semantic data as well, and that the findings of validation for the numeric data would in principle apply to semantic data.

Epistemological Differences Between Experimental, Statistical, and Logico-mathematic Designs

Two important questions that drive this paper are: Why did non metric structuralist theories and methods not become significant in psychology as they did for a time in other disciplines? And why did structuralist approaches fail to develop further in those disciplines? A simple response from an experimental and statistical perspective is that structuralist methods are not scientific. However, it is suggested here that the answers to these two questions lay in large measure within an epistemological gap between experimental and statistical and non metric logico-mathematic approaches.

Paradigm Shifting

It is suggested that one reason the logico-mathematic method and findings addressed here continue to meet with inappropriate critiques is that they engender a kind of paradigm shift and thus a shift in criteria for what constitutes a valid controlling methodology.[13] Obviously, the logico-mathematic method and its findings are sufficiently outside mainstream cognitive science such that the taken-for-granted background knowledge and concepts undergirding the method are not evident or transparent as they are with traditional methods. Though background knowledge is not necessary for carrying out the methodology, it does serve as a context for credibility assessment (somewhat like the "ground" in a figure–ground relationship defines the "figure").

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[13]I wish to thank David Smith for his conceptualizing the series of issues I had been addressing as constituting a paradigm shift.

Over the years, reviewers have charged the logico-mathematic method and its $S_{ub}L_{it}$ findings with making extraordinary claims. However, what constitutes an extraordinary claim is dependent on background knowledge. Something is considered an extraordinary claim only to the extent that it is not perceived as coherent with either accepted criteria for assessing validity or with other generally accepted knowledge.[14] With familiar methods, undergirding background knowledge is not an issue; it functions as a consensual "given." Belonging to the background knowledge required to accept the credibility of the logico-mathematic method and its findings is knowledge of structuralist epistemology. Lacking this, some of the seemingly anomalous methodological operations are perceived as not having even prima facie credibility. Some empirical findings for bridging this gap are presented in Haskell (in press).

Background knowledge pertinent to the logico-mathematic methodology include the cognitive research on metaphorical and analogical reasoning and mapping processes (e.g., Gentner, 1983; Haskell, 1987, 2002). Narrative stories, topics, and their $S_{ub}L_{it}$ referents can be conceptualized as unconscious "metaphorical" and "analogical" reasoning regarding discussants' interpersonal concerns about a narrative setting. In addition, $S_{ub}L_{it}$ narratives can be conceptualized as being a response to events in the narrative situation that function as "masked priming" stimuli (Haskell, in press). To understand the logico-mathematic method, then, requires an epistemological shift in what constitutes a valid methodology and in the knowledge base undergirding that method. Understanding the rationale for the extraordinary claims axiom, Haskell (in press) developed an extensive methodology to address the seemingly extraordinary claims assumed inherent in the logico-mathematic method.

Experimental and Logico-mathematic Knowledge

One conceptual shift that is required to understand the logico-mathematic methodology — in contrast to experimental and statistical methods — is that the latter tends largely to work with and to yield phenomenological realities, that is, everyday behaviors — whereas the former posits scientific reality *as that which is derived from operations performed on phenomenological realities or data*. Mathematical structures represent the extreme case of derived constructions performed on phenomenological data. It should be noted that much of mathematical psychology, however, is not an example of mathematical constructivism as it tends to be largely statistical.

[14]The concept of an "extraordinary claim," then, is a relative one. The claims established by the logico-mathematic method would likely not be considered extraordinary to structuralists like Piaget and Levi–Strauss. Indeed, some of the criticism leveled against Levi–Strauss is based on an epistemological gap similar to that being addressed.

Many — perhaps most — concepts in physics such as the gravitational constant, black holes, so-called worm holes and other (seemingly bizarre) mathematically derived structures are exemplifications of constructed (non phenomenological) knowledge, though certainly they are tied by some extensive chain of linkages embodied in complex mathematical equations to phenomenal data (in this regard see *[16.2] Nomological Validation Network*).[15] For Piaget, such constructivisitic operations result in assimilations that make it difficult to distinguish between observables that depend on these cognitive operations for their existence and direct observation of objects as phenomenologically perceived (e.g., the classic psychological problem as to what exactly constitutes a stimulus).

Certainly, findings based on correlations result from operations performed on data, but they are not of the same logical type as those derived structures from physics where mathematical operations performed on data suggest, for example, that *time* may be reversible, or that there is a gravitational constant yielding the rate of acceleration of bodies of lesser mass toward a more massive body. These scientific realities are neither phenomenally given, nor are they the consequence of a simple superimposing of mathematical concepts and procedures on to data, e.g., mapping of 1 + 1 = 2 onto peaches, or applying statistical procedures to data. They are derived and constructed from complex cognitive operations performed on data. That the earth is not the center of the solar system was not a perceptual given before photographs taken from outside the earth's orbit. The notion that the earth was not the center of the solar system was based on mathematical operations carried out by Kepler on Tycho Brahe's observational data. Recall that Kepler's third law states that the ratio of the cube of the mean distance of a planet from the sun to the square of its orbital period is a constant. Again, such physical realities are derived structures, not simple procedures performed on direct perception. This is quite different from performing an analysis of variance.

In this concrete sense, and contrary to conventional views, most applications of statistics in psychology tend to be phenomenological in that they work with, and are largely mapped onto, perceptually "given" concrete objects or behaviors; they do not generate abstract or constructed higher-order entities or realities that are law-like — though variables may be teased out of the data. In this sense, then, the objects of experimental and statistical methods tend to be based on folk concepts as Bruner (1990) and others like Churchland (1986) have noted. Such an approach tends to only yield additional phenomenological objects. Indeed, experimental and statistical methods so applied can be seen as a kind of experimental phenomenology. It is not

[15]The references in brackets, e.g., "*[16.2] Nomological Validation Network*" refer to sections in the Appendix of Haskell (2003, this journal).

being suggested that the level of analysis referred to as folk psychology is necessarily inappropriate. But contrary to what is generally believed, experimental and statistical methods are neither universal panaceas, nor methodologies for discovering constructive "laws."

Since the logico-mathematic design delineated in Haskell (2003, this journal) is more closely related to linguistics and mathematics than to experimental and statistical methods, accordingly, validation procedures also differ from experimental and statistical methods.[16] From a logico-mathematic perspective, both abstract realities and validation processes are based on a series of consistent *internal* and integral sets of transformation operations, with — as in mathematics — little *initial* regard for external (read: phenomenological) realities. Structurally, logico-mathematic validations are more like mathematical proofs. Later, however, such mathematical constructions are found to map onto external realities. For example, the internal structure of the exponential equation — like all un-applied mathematics — is independent of empirical reality. When applied, however, the exponential equation maps onto external references. For example, the exponential equation describes phenomena pertaining to the growth of populations, to the growth of egg production, or to the growth of knowledge. Similarly, the extended exemplifications of the set of numeric references in the narratives containing the number "3s" (see Haskell, 2003, this journal) show they are all variants of a set which form a permutation group, as well as a consistent set of transformations. Initially, as in mathematics, structures have only internal meaning, but later when contextually mapped onto the empirical narrative situation the external referent of their internal structure becomes evident.[17] To reiterate: structural method yields a reality *that is derived from operations performed on phenomenological data*.

It is clear that the logico-mathematic methodology and the cognitive processes of the researcher are reciprocal. This may seem contrary to the application of experimental and statistical procedures. But in fact, there is little difference: cognitive structuring and re(con)structuring are also inherent in applying experimental and statistical methods — unless a perceptually-based copy theory of objects along with operations performed on them, as well as

[16]I do not contend that the structure of the logico-mathematic method is a precise equivalent to mathematical structure, only that in important respects it formally parallels an integral logico-mathematic ordering structure. It is interesting to note, however, that of all reviewers of the logico-mathematic method over the years, of those who support it, one is a mathematician, the other a psychologist with a mathematics speciality.

[17]As to the ontological problem of how and why the "language" of mathematics, unlike ordinary language, is so successful in mapping on to external reality, a discussion would take this paper too far afield (but see Piaget, 1971a, pp. 333–345).

methodological manipulation procedures — are assumed to be somehow externally "given."[18]

Thus, while experimental procedures seem apparently "given," they are inextricably shaped by logico-mathematic-like cognitive operations to varying degrees. One view (e.g., Piaget, 1970) is that knowledge about an object or datum is the consequence of an assimilation into cognitive schemata which involves logico-mathematic-like operations. The congenitally blind who gain their sight do not initially "see" squares and triangles. These "objects" have to be cognitively constructed. The relation of logico-mathematics and experimental objects, then, is not simply that of higher-order operations superimposed on data, e.g., statistical operations, but rather that of a cognitive (con)structuring performed on data. This constructivistic view of reality is consistent with a weaker form of constructivism — but widely accepted — that holds perception and memory are constructed and are not strict copies of reality (see Neisser, 1976).

Finally, given that scientific concepts are not specifically attached to any field or methodology, procedures such as "control," "testing," and "falsification" are nevertheless typically defined and conceptualized exclusively in terms of an experimental and statistical paradigm and epistemology. Methods that seemingly do not easily conform to these domain-specific definitions are seen as invalid. The logico-mathematic method discussed here, however, has its own systematic control, testing, and falsification procedures comparable to more traditional methods.[19] Further, the logico-mathematic methodology is a more appropriate method for the analysis of language and for applying its systemic and integral validation procedures across various linguistic sets.

All areas of investigation, then, are not amenable to experimental design. Certainly linguistics is a clear case in point as is much of geology, chemistry, biology, and human evolutionary theory (until recent innovations in manipulating genes in the common house fly made it possible to manipulate and thus observe evolutionary change). Similarly, there are complex phenomena in psychology, such as narrative comprehension, that do not adequately yield to experimentation.

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[18]For a more extended explication, see Piaget's cognitive analysis (1971a, pp. 336–339).

[19]Experimental, statistical, and computational validation constructs can be applied to the logico-mathematic method. Briefly these include (a) face validity, (b) content validity, (c) construct validity, (d) external validity, (e) convergent validity, (f) discriminant validity, (g) concurrent validity, and (h) predictive validity. For example see *[14.] Matrix and Lattice Structure Validation Operations, [15.] Multicorrelative Transformational Validation Operations, [15.1.] Internal Order Structure Operations, [15.2.] External Order Structure Operations, [16.2] Nomological Validation Network, [16.3.] Falsification, [16.4.] Retrodiction*.

Experimental Design and $S_{ub}L_{it}$ Phenomena

In previous reviews, an over-arching criticism has been that $S_{ub}L_{it}$ phenomena need to be subjected to experimental design. While experimental and quasi-experimental designs can easily be applied, experimental designs are neither sufficient for establishing the validity of $S_{ub}L_{it}$ referents nor for teasing out linguistic complexity. Unfortunately, there have been only two experimental studies — albeit, peripheral ones — of sub-literal-like referents in verbal narratives. In these two early experiments, unconscious meaning was conceptualized as *unconscious projection* (Horwitz and Cartwright, 1952), and as group *projection of fantasies* (Farrell, 1979). A central point of this paper, however, is that even with experimental research on $S_{ub}L_{it}$ phenomena, an *a priori methodology is required for analyzing and validating the unconscious referents of a spoken narrative that the experiment is designed to manipulate*. Hence the further need for a method and theory independent of experimentation.

In applying an experimental approach to narratives, affect-arousing objects or situations could be introduced into a group of discussants as independent variables: a confederate is "planted" among discussants and instructed to act in a certain way; or a video camera is placed noticeably in the discussion room. Hypotheses could then be formulated about categories of $S_{ub}L_{it}$ topics (the dependent variable) that emerged in the discussion. Under conditions of a video camera, for example, it might be hypothesized that topics about "spying" would arise. In fact, such responses to the above variables have been observed and analyzed under quasi-experimental conditions (Haskell, 1982; Yalom, 1970).

As a control, a similar group of discussants not subject to the independent variables could be analyzed.[20] In addition, statistical analyses can be applied in terms of the analysis of variance, simple frequency, or correlations showing above chance levels for the appearance of particular topics. This would likely yield results showing that under conditions of no video camera or one-way mirror topics about spying do not appear at a statistically significant level. Under experimental conditions where positive correlations were found between the independent variables and "spy"-type narratives, however, the question would remain as to what the correlation means. Indeed, without a theory or framework on sub-literal meaning, it is difficult to understand why such an experiment would be conducted in the first place. Once a framework

[20]Subjects would have to be told that the experimenters were, say, professors of English, because the public's meaning of "psychologist" often carries with it a kind of surreptitiousness and therefore might itself generate topics of spying. It would be hypothesized that English professors, as an independent variable, would more likely generate topics about novelists. Researchers from different disciplines would likely activate different automatic stereotypes.

is accepted then the findings would at least have shown a positive correlation where none had been previously established in the experimental literature.[21] At the very least, experimental demonstration of the hypothesized correlations would require an explanation for the relationship and a call for further research.

Lawfulness based on an "Aristotelian" frequency approach, however, does not yield the "Galilean" invariant lawfulness that Lewin (1931) envisioned for behavioral explanations. Though correlations may establish the plausibility of sub-literal topics, they yield no explanation of them: establishing a correlation at a level of significance is not the same as establishing sub-literal meaning or reference for a topic — that is, that the topics are the direct consequence of the independent variable.[22] Such a correlation is only meaningful if one *implicitly* assumes an unconscious link between independent and dependent variables. This holds true for experimenting with conventional or literal reference in language as well; though being implicit, it is generally unrecognized.

Further, the "correlation problem" compounds itself in direct proportion to the increasing complexity of verbal reports. But there is yet another problem in applying experimental and statistical methods. Sub-literal meaning is typically yielded only by applying the intricate cognitive linguistic operations, e.g., phonological, semantic, syntactical, etc. These operations are not easily subjected to experimental and statistical methods — without having already assumed their $S_{ub}L_{it}$ referents. An additional, but not as significant a problem concerns researchers establishing and agreeing on what is meant by $S_{ub}L_{it}$-"type topics" generated in response to an independent variable, i.e., would resulting narratives about police be considered as belonging to the same category as spying references?

A strictly experimental approach often leads to increasingly finer grain analyses of variables. While for certain purposes breaking down findings into increasingly smaller components can be useful, when carried out on narratives the systemic structure and linguistic characteristics of a story and its linguistic operations are lost. Accordingly, only some elements of narrative analysis are amenable to experimental and statistical methods. Thus, while some aspects of narratives can be subjected to experimental, or quasi-experimental/statistical design, and provide augmentation to the basic hypothesis

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[21]I would like to thank my colleague David Livingstone Smith for pointing out that such findings would at least show a previously unrecognized *lawful* quantitative relationship between the independent variable or stimulus and the dependent variable or $S_{ub}L_{it}$ response where none had previously been shown experimentally.

[22]In linguistics the traditional distinction between "literal" and "figurative" meaning has become problematic. For purposes of this paper the term "literal" should be understood as "conventional" (see Haskell, in press).

regarding the existence of $S_{ub}L_{it}$ material, the analysis and validation requires a method that is congruent with the petinent properties of the subject matter.

Conclusion

This paper has comparatively examined the logico-mathematic methodology found in Haskell (2003, this journal) with experimental and statistical design. The paper has argued that there are significant epistemological differences between these two methodologies. It was suggested, too, that these differences require a paradigm shift for understanding the logico-mathematic method.

In comparing and contrasting the logico-mathematic approach with experimental and statistical methods, evidence, and theory, it has been pointed out that, despite their differences, in some ways the two approaches can also complement one another, with each method applicable to different aspects of $S_{ub}L_{it}$ phenomena. But in order for this to occur, the logico-mathematic method needs to be understood within its own terms.

Though no rigorous researcher would downplay the power of experimental and laboratory designs, the same generally can not be said of addressing the issue of increasing the robustness of experimental design with convergent data from everyday situations. Just as everyday phenomena need to be subjected to experimental testing, so do experimental designs need to be informed by the conditions attached to everyday phenomena for which they serve as models. No aircraft flight design is based solely on findings from wind tunnel experiments but on in-flight data from similar aircraft and from early prototypes of actual aircraft. Similarly, laboratory findings in psychology can be increasingly informed with variables and situations closely resembling those operations in everyday settings.[23]

[23]A following paper (Haskell, in press) addresses theoretical issues undergirding unconscious cognition and dual meaning, presents corroborating evidence for two of the more controversial operations, suggests a biological evolutionary base, and presents further implications of the logico-mathematic method and its findings.

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