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Research Overview

One focus of my current research in **applied analysis** is on convergence phenomena for a class of orthogonal polynomials in the complex plane and related probability measures associated with discrete-time random processes. This work was initially motivated by problems in spectral analysis, digital filtering and linear prediction, and is appealing since one is able to bring to bear the strength and elegance of real, complex, harmonic, and functional analysis, and also because of its applied nature.

Given a measure, μ, on the unit circle, the *Szegö* polynomial of degree k with respect to μ, which we denote $P_k(z,\mu)$, is the monic polynomial which minimizes the L^2 norm $\int_{-\pi}^{\pi} |p(e^{i\theta})|^2 \, d\mu(\theta)$ over all monic polynomials of degree k. The Szegö polynomials with respect to a measure form an orthogonal family; multiples of those obtained by Gram-Schmidt.

The study of Szegö polynomials dates to the work of Wiener, Levinson, Geronimus, and Granander and Szegö, but has also appeared in the literature in the last decade. One general question I am interested in is that of the behavior of the family $\{P_k(z,\mu_h)\}_{h\to 0}$ where k is fixed and the measures μ_h converges to point masses:

$$\lim_{h\to 0} \mu_h = \mu_\delta := \sum_{j=1}^{m} \alpha_j \delta_{\theta_j}, \tag{1}$$

where the α_j are complex numbers and the convergence is in the weak-star topology on measures. For $k > m$, $P_k(z,\mu_\delta)$ is not uniquely defined, and the $P_k(z,\mu_h)$ need not approach a (unique) limit, though any limit point will have zeros at the $e^{i\theta_j}$. My recent results explicitly characterize (unique) polynomial limits when $\mu_h = \mu_\delta + h\gamma$, where γ is an arbitrary absolutely continuous measure, and when μ_h is formed by convolving μ_δ with either the Poisson or the Fejér kernel. Point masses are the spectral measures associated with sinusoidal signals, and the phases of the zeros of Szegö polynomials are used as frequency estimates. Some of my results illuminate recent work and proposed methods in frequency estimation [1, 3, 4, 5] which have addressed asymptotic behavior of Szegö polynomial zeros, non-existence of limits, and distinguishing "signal Zeros" from "extraneous zeros", for $k > m$.

Current investigations include characterization of Szegö polynomial limits for convolution of μ_δ with any of a class of approximate identities of the form $\sum a_n \phi_n(\theta)$ where $\sum a_n = 1$ and $\phi_n(\theta)$ is the Fejér kernel. It can be shown that the Poisson kernel is of this form. One goal of this research is to is to use polynomial limits to define equivalence classes of kernels, and to characterize each class. I am also interested in characterizing measures for which (1) holds which give rise to non-unique Szegö polynomial limits. I give a construction in a paper recently submitted to the Journal of Math. Analysis and Applications of a sequence of strongly convergent measures which give rise to an infinite number of polynomial limit points. I am also interested in the *reflection coefficients*, or constant terms, $P_k(0,\mu_h)$ as both $h \to 0$ and polynomial degree $k \to \infty$ in a prescribed manner. The asymptotic behavior of reflection coefficients gives information about the region of accumulation of polynomial zeros. I have obtained a preliminary result for a special case of $\lim_{k\to\infty} \lim_{h\to 0} P_k(0, \mu_h)$. Observed phenomena regarding polynomial zeros as $k \to \infty$ in the electrical engineering literature was described mathematically in [2].

Another focus of my research is the work undertaken for the **Army Research Laboratory**, Adelphi, Maryland, where *adaptive* signal processing techniques are used for attenuation of broad band noise in acoustic arrays, either ground based, or mounted on vehicles. Adaptive techniques are used in signal processing when noise statistics are either unknown or changing, or both, as is typically the case. Existing methods, particularly those based on the least-mean-square algorithm, have been successful in attenuating high-power narrow-band interference. However, broadband components of lesser power are not significantly attenuated. I have implemented a new algorithm which first cancels the narrow-band interference. This approach is shown to reduce broadband noise in simulations, however, it does not perform well on actual test data due to low correlation between the *primary* sensor, which contains the target, and the *reference* sensor, which is used to sense the noise. A proprietary technique, proposed by a private-sector company, is currently being developed and evaluated. The computational work is performed with Matlab. This work has also afforded me the opportunity to work and interacted with engineers in government and industry, and has given me a broader perspective of applied mathematics.

Some aspects of my research lend themselves to simulation and experimentation at the level of **undergraduate mathematics**. The Szegö polynomials are computable as a ratio of matrix determinants whose elements are Fourier coefficients of a probability density. With the aid of Matlab, for example, computer experiments can be designed that encourage conjecture, for example, about the behavior of polynomial zeros observed in simulations. Much of the analytical work could then be done using the methods of linear algebra and elementary complex variables. Similarly, many adaptive filtering algorithms, for noise cancellation and other applications, can also be understood with knowledge of linear algebra and complex variables. Matlab is especially well-suited to simulation and algorithm design, and also for importing real-world data. I hope to continue to look to my research relationship with ARL as a source of opportunities for undergraduate research.

References

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